

Note

**An Improved Form of the Artificial
Diffusion Parameter- χ**

INTRODUCTION

The artificial diffusion mechanisms (e.g., the Shuman switch [1], the hybrid scheme [2]) used to damp the post shock oscillations or to stabilize the computations contain a factor χ which determines the amount of diffusion provided. The value of χ is required to be selected based on trial calculations and usually the exhibition of a sharp shock profile is considered as the criterion. A criterion like this seems to be adequate for a simple flow as that of a flow behind a shock. But when the flow exhibits some complexity, e.g., a shock tube flow, such a selection of χ poses problems. A complex flow field involves a marked variation of the Courant number giving rise to a variation of truncation error from point to point and hence a variation in the dispersion error. Thus, χ based on any one criterion may prove to be excessive in certain regions. For example, the value of χ which ensures a smooth shock profile in a shock tube flow may cause an excessively smeared profile of the contact discontinuity. On the other hand, a value of χ which ensures a sharp profile of the contact discontinuity may not fully damp the post shock oscillations. A remedy to overcome this drawback seems to be as follows.

- (1) to vary χ from point to point depending upon the local Courant number; and
- (2) to employ a value of local χ_i which is determined from a monotonicity analysis.

In this note, we compare the results obtained by employing a locally dependent value of χ with those for a global value of χ . The second order MacCormack scheme [3, 4] is used as the basic scheme.

VALUE OF χ

The linear version of the Shuman switch incorporated MacCormack scheme [1] or that of the hybrid scheme [2] is

$$\begin{aligned}
 W_i^{n+1} = & W_i^n - \frac{\sigma}{2} (W_{i-1}^n - W_{i+1}^n) \div \frac{\sigma^2}{2} (W_{i-1}^n - 2W_i^n + W_{i+1}^n) \\
 & + \frac{\chi}{2} (W_{i+1}^n - 2W_i^n + W_{i-1}^n)
 \end{aligned}
 \tag{1}$$

where the symbols have their usual meaning and σ is the Courant number.

A monotonicity analysis following [5] shows that the above scheme is monotonic for all

$$\chi \geq \sigma(1 - \sigma). \quad (2)$$

The factor $\sigma(1 - \sigma)$ has been used with Van Leer's monotonicity operator [7].

RESULTS AND DISCUSSION

The scheme used for the present computations is

$$W_i^{n+1} = L_{MC} W_i^n + \frac{1}{2} \{ \theta_{i+1/2} (W_{i+1}^n - W_i^n) - \theta_{i-1/2} (W_i^n - W_{i-1}^n) \}$$

with

$$\theta_{i+1/2} = \chi_{i+1/2} \left\{ \frac{|P_{i+1} - P_i|}{|P_{k+1} - P_k|_{\max}} \right\}^m$$

and

$$\chi_{i+\frac{1}{2}} = |\sigma_i| (1 - |\sigma_i|), \quad m \geq 1.0, \quad \sigma < 1.0 \quad (3)$$

L_{MC} = operator of the MacCormack scheme.

P = any flow variable (density in the present computations).

Thus we are employing a minimum value of χ_i that ensures monotonicity locally. The scheme 3 could be considered as a hybrid of the schemes L1 and L2 defined as,

$$L_2 = L_{MC}, \quad (4)$$

$$L_1 W_i^n = L_{MC} W_i^n + \frac{1}{2} [\chi_{i+1/2} (W_{i+1}^n - W_i^n) - \chi_{i-1/2} (W_i^n - W_{i-1}^n)].$$

The test problems and the initial conditions employed are those described in [6]. Table 1 compares the results obtained by the use of the Shuman switch with a global χ with those for the scheme 3. (It was verified by extensive experimentation by the present authors that the behaviours of the Shuman switch and the scheme 3 are identical given the value of χ .) The flow profiles are qualitatively similar to those obtained with the Shuman switch incorporated schemes described in [6] and are not given here. Values of $m = 1$ and 2 were tried in the present experiments. The results with $m = 2$, were acceptable only in certain of the cases considered. Hence, only the results with $m = 1.0$ are discussed in the note. An examination of the table shows that the scheme 3 generally gives a sharper profile of the shock or the contact discontinuity or that of density near the wall than the Shuman switch with a global χ . But the scheme 3 leaves pressure overshoots undamped for the reflected shock. This may be due to insufficiency of the local monotonicity analysis or due to the form of θ used. The contact discontinuity profiles are largely smeared due to a locally linear behaviour of equations in the neighbourhood of the contact discontinuities. It is to be

TABLE I

Comparison of the Results Obtained with Scheme 3 (local χ) with Those of the Shuman Switch (global χ)

Test problem	Shuman switch				Scheme 3, $m = 1.0$	
	$\sigma = 0.7$		$\sigma = 1.0$		$\sigma = 0.7$	$\sigma = 1.0$
	$\chi = 1.0$	$\chi = 2.0$	$\chi = 1.0$	$\chi = 2.0$		
<i>(1) Propagating shock</i>						
Smearing (No. of mesh widths)	6	9	6	9	5	4
Pressure overshoot (%)	2.58	1.71	—	—	1.92	—
Integrated error (relative) ^a	1.33	2.24	2.58	4.45	0.83	1.08
<i>(2) Reflected shock</i>						
Smearing (No. of mesh widths)	5	7	7	10	4	3
Pressure overshoot (%)	0.3	—	1.2	0.17	3.10	9.3
Integrated error (relative)	0.8	1.25	1.44	2.17	0.511	0.46
Density departure at the wall (%) ^a	-6.43	-7.45	-7.72	-9.65	-3.19	-0.735
No. of meshes to theoretical density ^a	10	13	>12	>12	6	steep rise
<i>(3) Standing shock</i>						
Smearing (No. of mesh widths)	13	20	10	15	6	6
Pressure overshoot (%)	—	—	—	—	0.954	1.22
Integrated error (relative)	1.48	2.41	1.3	1.71	0.68	0.54
<i>(4) Shock tube flow, shock</i>						
Smearing (No. of mesh widths)	10	14	10	15	5	4
Pressure overshoot (%)	3.7	—	3.2	—	2.0	2.0
<i>Contact discontinuity</i>						
Smearing (No. of mesh widths)	15	20	20	25	15	17

^a For the definition of these terms see [6].

noted that the Shuman switch behaves unsatisfactorily for the shock tube problem, giving largely smeared profiles of the shock and contact discontinuity. It was noticed with the Shuman switch that any effort to reduce the smearing of any one feature in the shock tube flow would lead to an incomplete damping of oscillations near the other feature. Local variation of χ seems to be quite successful in overcoming this drawback. The advantages in employing a local χ_i in place of a global χ with the Shuman switch or the hybrid scheme is thus clearly demonstrated. One could also use a scheme of the form 3 and obtain similar results.

There is one drawback in the use of $\chi_i = \sigma_i(1 - \sigma_i)$, namely, that it is ineffective in regions where $\sigma_i = 1.0$. A better form of θ as used in [7] can be employed to overcome this drawback. We wish to emphasize that the mechanisms discussed here are not necessarily the best ones for the purpose, but only that a local variation of χ is to be preferred to the usage of a global χ .

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